MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 1, 2018/2019**

## PEM0036 – CALCULUS

( Foundation in Engineering )

27 OCTOBER 2018
9.00AM – 11.00AM
( 2 Hours )

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**INSTRUCTIONS TO STUDENT**

1. This Question paper consists of 3 pages, excluding the cover page and appendices.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided. All necessary working **MUST** be shown.

4. Only **NON-PROGRAMMABLE** calculator is allowed.

**Please answer ALL questions and show the necessary working. Total mark is equal to 100.**

## Question 1

a) Find the following limits:

(i) $\lim_{x\to\frac{1}{2}} \dfrac{2x-1}{6x-3}$ (3 marks)

(ii) $\lim_{m\to 5} \dfrac{\frac{1}{m}-1}{m-1}$ (4 marks)

(iii) $\lim_{x\to\infty} \dfrac{3x^2+15x+12}{2x^2+3x-2}$ (4 marks)

(iv) $\lim_{n\to 0} \dfrac{\sqrt{n+9}-3}{n}$ (6 marks)

b) Determine whether the function, $f(x)$ is continuous or not. (4 marks)

$$f(x)=\begin{cases}\sqrt{8-2x} & ,x<2\\ 4-x & ,x=2\\ 10-2x^2 & ,x>2\end{cases}$$

c) Find the vertical and horizontal asymptotes of $g(x)=\dfrac{x^2-1}{x^2-x-2}$. (4 marks)

**[25 marks]**

**Continued ...**

## Question 2

a) Show that the derivative of function $F(x) = \dfrac{1-\tan x}{1+\tan x}$ is

$F'(x) = -\dfrac{2}{(\cos x + \sin x)^2}$. (7 marks)

b) Given $x + xy - 2x^3 = 2$. Find $\dfrac{dy}{dx}$ by differentiating implicitly. (4 marks)

c) $J(m,n)$ is the minimum point of the curve $y = 3x^3 - 3x^2 - 36x + 5$. Find the values of $m$ and $n$. [Leave your final answer in 2 decimal places] (10 marks)

d) Given that $p = 4q + \dfrac{2}{q}$, find the rate of change of $p$ when $q = -1$ if $q$ increases at 2 units per second. (4 marks)

**[25 marks]**

**Continued ...**

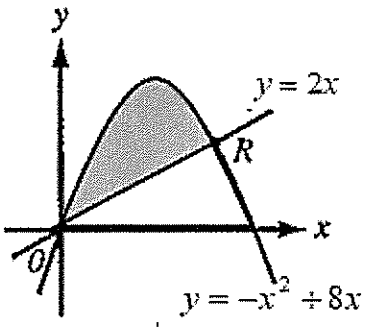
## Question 3

a) Integrate $\int (3x-1)\, e^{-x} dx$. (3 marks)

b) Evaluate the integral $\int_3^4 \frac{2x+5}{\sqrt{x-3}} dx$. (8 marks)

c) Evaluate the integral $\int \frac{6x+7}{(x+2)^2} dx$. (7 marks)

d) The diagram below shows the shaded region bounded by the curve $y = -x^2 + 8x$ and the straight line $y = 2x$. The curve intersects the line at origin and point $R$.




Find the area of the shaded region. (7 marks)

**[25 marks]**

## Question 4

a) Solve the differential equation $\frac{dy}{dx} = e^{3x+2y}$ by using separation of variables. (6 marks)

b) Find the solution of the initial value problem $y'' + y = 0$ if $y(0) = 3$ and $y\left(\frac{\pi}{2}\right) = -3$. (7 marks)

c) Find the general solution of the differential equation $y''+5y'+6y = 2x$. (12 marks)

**[25 marks]**

**End of Page.**

# APPENDICES

**Quadratic formula:** $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Derivatives:** $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

## Differentiation Rules

### *General Formulae*

1. $\dfrac{d}{dx}[u \cdot v] = u\dfrac{dv}{dx} + v \cdot \dfrac{du}{dx}$
2. $\dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
3. $\dfrac{d}{dx}(x^n) = nx^{n-1}$
4. $\dfrac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$

### *Exponential and Logarithmic Functions*

1. $\dfrac{d}{dx}(e^x) = e^x$
2. $\dfrac{d}{dx}(a^x) = a^x \ln a$
3. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x \ln a}$

### *Trigonometric Functions*

1. $\dfrac{d}{dx}(\sin x) = \cos x$
2. $\dfrac{d}{dx}(\cos x) = -\sin x$
3. $\dfrac{d}{dx}(\tan x) = \sec^2 x$
4. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
6. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

## Table of Integrals

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$

## Application of Integrals:

*Areas between Curve,* $A = \int_a^b [f(x) - g(x)]\,dx$

$Volume\ (Disk) = \pi \int_a^b [f(x)]^2\ dx$

$Volume\ (Washer) = \pi \int_a^b [f(x)]^2 - [g(x)]^2\ dx$

$Volume\ (Cylindrical\ Shells) = \int_a^b 2\pi (shell\ radius)(shell\ height)\ dx$

## Differential Equations

### *Linear Differential Equations*

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

### *Constant Coefficient of Homogeneous Equations*

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### *General Solutions to the Auxiliary Equation:*

| | |
|---|---|
| Two Real & Unequal Roots ($b^2 - 4ac > 0$) | $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ |
| Repeated Roots ($b^2 - 4ac = 0$) | $y = c_1 e^{rx} + c_2 x e^{rx}$ |
| Two Complex Roots ($b^2 - 4ac < 0$) | $y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$ |

### *Constant Coefficient of Non-Homogeneous Equations*

$y = y_c + y_p$ [$y_c$: complementary solution, $y_p$: particular solution]